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Sub-wavelength resolution of optical fields probed by single trapped ions: Interference, phase modulation, and which-way information

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Taking recent experiments as examples, we discuss the conditions for sub-wavelength probing of optical field structures by single trapped atoms. We calculate the achievable resolution, highlighting its connection to the fringe visibility in an interference experiment. We show that seemingly different physical pictures, such as spatial averaging, phase modulation, and which-way information, describe the situation equally and lead to identical results. The connection to Bohr's moving slit experiment is pointed out.

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INTRODUCTION

In several recent experiments, single trapped ions have been used to map optical fields, and a resolution considerably below the wavelength has been reported [1, 2, 3]. In the first of these experiments [1], a part of the resonance fluorescence of a single Ba^+ ion was back-reflected onto the ion with a distant mirror, and the resulting emission was found to be modulated upon variation of the ion-mirror distance. In the second experiment [2], a single Ca^+ ion was placed inside an optical cavity, and light coupled into the cavity was resonantly scattered. When the ion was shifted along the cavity axis, periodic variation of the emission was observed. In the most recent experiment [3], the setup was similar but here the cavity was resonant with an electric-quadrupole transition in Ca^+. This transition was coherently driven by light coupled into the cavity, and the excitation probability was found to be strongly modulated with the position of the ion in the cavity mode.

All these studies are based on the interaction of a single atom with an optical standing wave. While in the two Ca^+ experiments a standing wave of resonant light forms inside the optical cavity, in the Ba^+ experiment part of the electromagnetic mode structure (or vacuum field) around the ion is transformed into a standing wave by the back-reflecting mirror.

Sub-wavelength resolution is achieved because the position of the ion relative to the mirror(s) is well-controlled and the ion's spatial wavefunction is confined to a region much smaller than the optical wavelength λ (400 to 800 nm). This strong confinement is due to the trapping potential of a Paul-type ion trap, which to very good approximation can be considered harmonic with typical oscillation frequencies Ω_t from 1 to several MHz and a spatial extension of the lowest energy eigenstate around 10 nm. Laser cooling can prepare ions in thermal states with low average quantum numbers. Preparation of the motional ground state with high purity has also been achieved by means of sideband laser cooling [4, 5, 6]. In the cases considered here, the ions were Doppler-cooled, i.e. their motional energy E_{th} was comparable to the linewidth of the cooling transition which in all cases is about 20 MHz. Since the spatial extension scales with $\sqrt{\bar{n}}$, where $\bar{n} = E_{th}/\hbar\Omega_t$ is the thermal energy in units of trap quanta, a resolution between about 10 and 50 nm was achieved.

With this resolution, local variations of an optical field can be detected by shifting the single ion through the field structure. Therefore this technique received the name optical nanoscope [2]. Similar sub-wavelength mapping techniques are used in microscopy [7], and they have also been demonstrated with single molecules instead of ions [8].

While the typical length scale of an optical field is its wavelength, smaller structures can emerge, e.g., in diffraction patterns, as high-order modes of optical resonators, or in general when several partial waves are superimposed and interfere. A standing wave is a comparatively simple structure, formed by superposition of two counterpropagating travelling waves of equal amplitude and polarization. It is nevertheless a highly instructive case because the observation of a standing-wave structure is connected to the interference between two processes pertaining to the two travelling waves. The resolution with which the structure is detected determines the visibility of the observed interference fringes. In turn, the observed visibility is a measure for the resolution and hence for the spatial extension of the ion as well as for its thermal energy.

On the other hand, a limited visibility in an interference measurement may be a signature for the presence of which-way information which in principle can be extracted from the system. In our cases, the which-way information must be stored in the motional degrees of freedom of the ion, because it is the motion in the trap which determines the spatial size and shape of the wave packet of the ion.

The purpose of this paper is to present and compare these various physical pictures and to show how thermal motion of the ion, resolution, visibility of interference fringes, and which-way information are related. We will apply these ideas to analyse the three mentioned experiments and compare the experimental results. We will give classical and quantum descriptions of the situations and show that, although their interpretations look rather different, they are equally valid and lead to the same conclusions.

We focus here on the connection between ion motion, interference, and resolution of the field structure. The question how the presence of a distant mirror affects the *internal* dynamics of the ion has been discussed with a quantum model in Ref. [9].

INTERFERENCE AND VISIBILITY

First let us explain how in the three cases the observation of the standing wave can be interpreted as an interference between two processes. In the first experiment, a laser excites the ion from one side, and photons scattered under 90° are detected, see Fig. 1a. A mirror is placed on the opposite side (at −90°), such that photons scattered into that direction are back-reflected and also sent into the detector. Clearly the two pathways into the detector are indistinguishable and interfere, which is observed as a modulation of the detector signal (photon count rate) vs. the distance between mirror and ion [1]. In the second experiment, where light from a cavity mode is scattered by an ion [2], two scattering amplitudes corresponding to the two counterpropagating waves contribute to the detection of a photon, see Fig. 1b. Depending on the ion's position between the cavity mirrors, these amplitudes are superimposed in or out of phase, thus interfering constructively or destructively. The same explanation holds for the cavity-induced quadrupole excitation [3], only that here the two excitation pathways into the long-lived upper state interfere, rather than two scattering amplitudes.

A point-like probe would be able to map the interference fringes with perfect visibility, but with a real atom the visibility will always be smaller. In the qualitative discussion above, we have already used one of the possible physical pictures for this visibility reduction: The spatial wave packet of the ion probing the optical field structure acts as an apparatus function with which the ideal signal (of a point-like probe) has to be convoluted to find the experimental signal. This particular "spatial" picture applies most intuitively to the cavity experiments. Taking a "temporal" view, the two processes which interfere on the detector may indeed have happened (at the ion) with a delay between them, during which the ion has moved. This is certainly true for the mirror experiment where one partial wave is delayed by the time it takes to the mirror and back, about 1.7 ns. The visibility will be reduced because the two interfering pathways do not find the ion in exactly the same state. There is also a "spectral" explanation for visibility reduction: The ion oscillates in the trap so that, depending on the momentary Doppler shift, it sees (or scatters) the two travelling waves with different spectra. Only the overlapping spectral components can interfere.

Finally, there is a which-way interpretation. It takes into account that a scattered or absorbed photon has a mechanical effect on the ion. On average, every scattering event will leave one photon recoil in the atom, such that its motional state may change in the course of the process. This recoil, however, encodes with which of the two travelling waves the ion has interacted, and inasmuch as this information is stored in the ion, the visibility of the interference will be diminished.

We will now develop these different pictures in detail and compare the conclusions to which they lead.

ION AS APPARATUS FUNCTION

When we treat the wave packet of the ion as an effective apparatus function, the observed signal is calculated as follows. Let x denote the spatial coordinate along the optical axis. The ideal signal from a point-like atom probing the standing wave would be

$$S_{ideal}(x) = 2\bar{S}\cos^2(kx) = \bar{S}\big(1+\cos(2kx)\big) \ , \qquad (1)$$

where $\bar{S}$ is the average signal and $k = 2\pi/\lambda$ is the wave vector of the light. Now let $\rho(x-x_0)$ be the probability to find the ion at position x when the trap center is at x_0. Then the observed signal as a function of the trap position x_0 is

$$S(x_0) = \bar{S}\big(1 + V\cos(2kx_0)\big) \ , \qquad (2)$$

where

$$V = \int_{-\infty}^{\infty} dx\ \rho(x)\cos(2kx) \qquad (3)$$

is the visibility of the interference fringes, in agreement with the standard definition $V = (S_{max} - S_{min})/(S_{max} + S_{min})$. We have used that ρ is a symmetric function, which is certainly true for a harmonic trap. In fact, as we will show later, in all cases which we treat here ρ is a Gaussian,

$$\rho(x) = \frac{1}{\sigma\sqrt{2\pi}}\ \exp(-\frac{x^2}{2\sigma^2}) \qquad (4)$$

with rms spatial extension $\sigma = \sqrt{\int_{-\infty}^{\infty} dx\ x^2\rho(x)}$. In this case the visibility according to Eq. (3) is

$$V = \exp(-2(k\sigma)^2) \ . \qquad (5)$$

Relation (5) is used to determine the resolution from a measured visibility. The derived value of σ will be

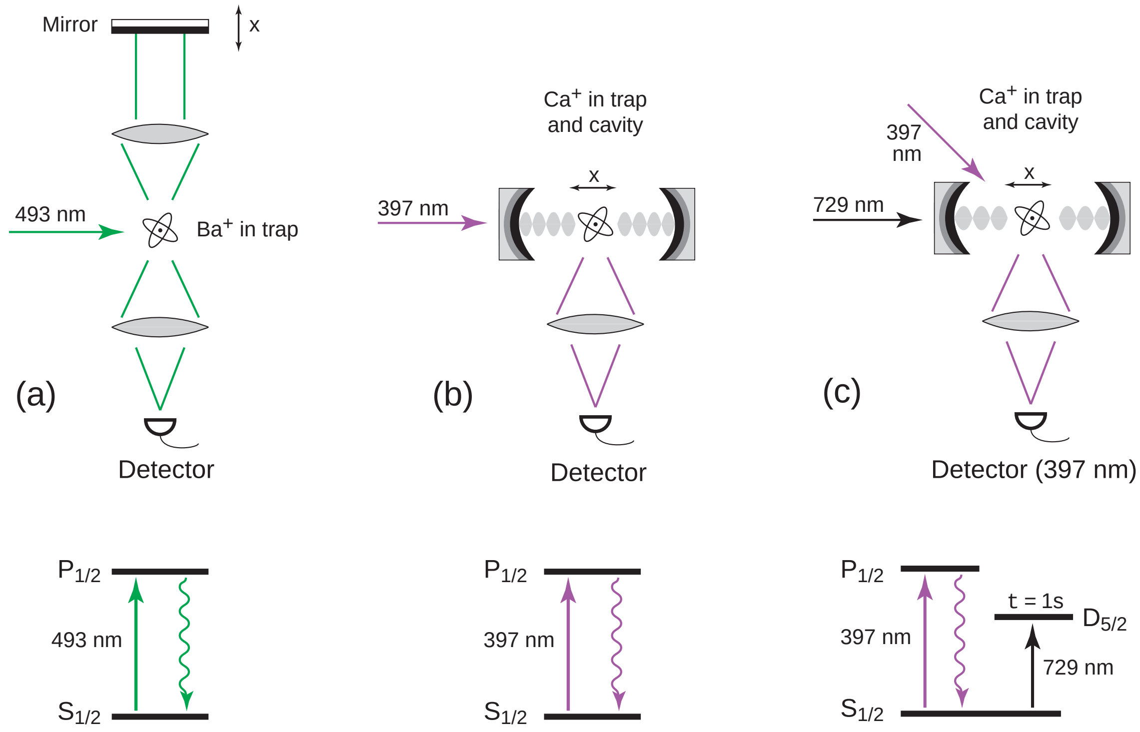

FIG. 1: Schematics of the three experiments. (a) Back-reflection experiment [1] with Ba^+, relevant levels and laser wavelength. (b) Setup with ion scattering cavity light [2], relevant levels of Ca^+ and laser wavelength. (c) Setup for quadrupole excitation by cavity light [3], relevant levels and lasers; after a pulse of cavity light (729 nm), the probability for excitation into $D_{5/2}$ is measured through state-selective fluorescence on the $S_{1/2}$ to $P_{1/2}$ transition (397 nm). In all experiments the typical measurement time is much larger than the oscillation period of the ion in the trap, Ω_t^{-1}.

an upper limit for the true rms spatial extension of the ion, as other broadening effects may be present in the experiment. The values for the three experiments, as calculated from the measured visibility values $V = 72\%$ [1], 40% [2], and 96.3% [3], are 32 nm, 43 nm, and 16 nm, respectively. It should be noted that in Ref. [1] the actual wave packet size is estimated to be 21 nm [10], and the reduced visibility is partly due to optical aberrations. It should also be mentioned that the larger number given in Ref. [2], 60 nm, is based on a different definition (by a factor $\sqrt{2}$) of the spatial extension [11] and is consistent with our result.

DOPPLER EFFECT

In the case of the ion in front of a mirror, another description is particularly intuitive which accounts for the time-dependent Doppler effect of the ion oscillating in the trap. This view highlights the role of both the delay between the two partial waves before they reach the detector, and of the spectral effect of the ion's motion.

First we assume the ion to be oscillating classically with frequency Ω_t and amplitude x_c, i.e. its momentary position is $x(t) = x_0 + x_c \sin(\Omega_t t)$. The oscillation modulates the phases of the two partial waves $E_\pm$ emitted towards the detector (one directly, the other via the mirror) according to

$$E_\pm(t) = E_0 \, e^{i(kx_0 \pm kx_c \sin(\Omega_t t) - \omega t)} \, . \qquad (6)$$

These two fields reach the detector with a phase delay e^{2ikL} between them, where L is the distance between trap center and mirror. The resulting detector signal is

$$S_c = |E_0|^2 \langle |e^{ikx_c \sin(\Omega_t t)} + e^{2ikL} e^{-ikx_c \sin(\Omega_t t)}|^2 \rangle \, , \qquad (7)$$

where $\langle\ \rangle$ denotes time averaging over many periods of the trap oscillation. Such an integration time $T \gg \Omega_t^{-1}$ is used in all the experiments discussed and will be assumed throughout our considerations. From Eq. (7) we get the visibility reduction due to sinusoidal oscillation

$$S_c = \bar{S} \, \left(1 + J_0(kx_c) \cos(2kL)\right) \, , \qquad (8)$$

where J_0 is the zero-order Bessel function.

A laser-cooled ion is not oscillating classically but in a thermal state, i.e. its oscillation amplitude x_c follows a thermal probability distribution. This distribution is derived from the Boltzmann distribution for the ion's energy and is given by

$$P(x_c) \, dx_c = \frac{x_c}{\sigma^2} \, \exp(-\frac{x_c^2}{2\sigma^2}) \, dx_c \, , \qquad (9)$$

where σ is the rms spatial extension as before, related to the thermal energy by $E_{th} = M\Omega_t^2\sigma^2$ with ion mass M. Combining Eqs. (8) and (9) we get for the detector signal

$$S(L) = \bar{S}\ \left(1 + \exp(-2(k\sigma)^2)\cos(2kL)\right) \qquad (10)$$

in agreement with the previous result, Eq. (5). One finds the same result when one evaluates first the spatial probability distribution for the classical oscillator, $P_c(x) = (\pi\sqrt{x_c^2 - x^2})^{-1}$ and integrates it with distribution (9), which yields the Gaussian of Eq. (4).

The picture of a Doppler effect is equally valid for the light scattering from a cavity mode. In this case the phase delay e^{2ikL} between the two partial waves in Eq. (7) is replaced by the relative phase between the two counterpropagating waves in the cavity, which varies as e^{2ikx_0} with the position x_0 of the trap center in the standing wave ($x_0 = 0$ is set to an antinode). In the same manner it applies to the cavity-induced excitation. There Eq. (7) is interpreted as the excitation probability when atomic saturation effects are neglected, c.f. Ref. [3].

Thus the phase modulation through the Doppler effect and the spatial apparatus function are in fact only different pictures for the same situation, yielding in all cases the same results for the visibility and the resolution.

WHICH-WAY INFORMATION

The relation between fringe visibility and which-way information in an interference experiment is at the core of wave-particle duality. It has been the subject of several general studies [12], and it was recently studied in experiments with atom interferometers [13, 14].

With a trapped ion probing an optical field, the encoding of which-way information happens through the recoil of an absorbed or an emitted photon. To illustrate this, we will use the example of Fig. 1b where an ion scatters cavity light; later in this section we will show that the same description applies to the other two experiments.

First assume that before scattering the ion is at rest. Depending on the travelling wave from which a photon is absorbed, the photon recoil will leave the ion oscillating with a certain initial momentum, i.e. a certain phase. It is this phase which carries the which-way information. The second part of the scattering process, the emission of the photon into the detector, will always leave the same recoil kick and not introduce any further distinguishability.

Now if every absorbed photon kicked the atom, the two final states pertaining to the two travelling waves would always be different, and there would be no interference. This is the extreme case of a very shallow trap ($\Omega_t \to 0$), where the two possible recoil momenta accelerate the ion to opposite sides, such that the two processes could be distinguished with certainty. Because of the trapping potential, however, a certain fraction of all absorption processes will leave the motional state unchanged. This is the so-called Lamb-Dicke effect [15], an important concept in laser cooling of trapped atoms [16]. It is this fraction of scattering events which creates the interference.

In more detail, the recoil of the two travelling waves is transferred to the ion's motional state by the spatial part of the respective electric field operators, $\mathrm{e}^{\pm ikx}$ [17]. An initial energy eigenstate $|n\rangle$ is thereby transformed into a superposition of states according to

$$|n\rangle \ \to \ \mathrm{e}^{\pm ikx}|n\rangle \ . \qquad (11)$$

The overlap $\langle n|\mathrm{e}^{2ikx}|n\rangle$ of these two possible final states is expected to determine the visibility of the interference.

Using Eq. (11), the rate at which an ion would scatter photons from one single travelling wave into the detector is given by [18]

$$S_{RW} = S_{rest}\sum_n P(n)\sum_k |\langle k|\mathrm{e}^{\pm ikx}|n\rangle|^2 \ , \qquad (12)$$

where S_{rest} is the scattering rate for an ion at rest, and $P(n) = \bar{n}^n/(\bar{n}+1)^{n+1}$ is a Boltzmann distribution over the harmonic oscillator states [19]. For a standing wave, the matrix element in Eq. (12) is replaced by $\langle k|2\cos(k(x-x_0))|n\rangle$, where x_0 is the position of the ion relative to an antinode as before. This yields for the scattering rate from the standing wave, as a function of the ion's position,

$$S(x_0) = 2S_{rest}\Big(1 + \sum_n P(n)\langle n|\cos(2k(x-x_0))|n\rangle\Big) \ . \qquad (13)$$

Since the spatial eigenfunctions $|n\rangle$ have definite parity, and using $2S_{rest} = \bar{S}$, we get the interference signal

$$S(x_0) = \bar{S}\big(1 + V\cos(2kx_0)\big) \ , \qquad (14)$$

where the visibility is given by

$$V = \sum_n P(n)\langle n|\cos(2kx)|n\rangle \ . \qquad (15)$$

Since $\langle n|\cos(2kx)|n\rangle = \langle n|\mathrm{e}^{2ikx}|n\rangle$, we find that the visibility of the interference fringes is indeed equal to the (thermally averaged) overlap of the two possible final states $\mathrm{e}^{\pm ikx}|n\rangle$ of the individual processes, just as the which-way interpretation suggests [20].

The same arguments are readily applied to the other two experiments: In the case of Fig. 1c, the photon recoil enters in exactly the same way, only the signal $S(x_0)$ describes the position-dependent transition probability into the upper state. In the case displayed in Fig. 1a, it is the recoil of the *emitted* photons which encodes the which-way information according to their direction of emission, while absorption always happens from the same travelling-wave laser beam and has no further effect.

Finally, we can evaluate Eq. (15) using the properties of the harmonic oscillator eigenfunctions [21], and we find

$$V = \exp(-2(k\sigma)^2) \tag{16}$$

where σ is again the rms spatial extension of the ion, now calculated from the thermal distribution over the quantum states, $\sigma^2 = \sum_n P(n)\langle n|x^2|n\rangle = (2\bar{n}+1)\langle 0|x^2|0\rangle$.

Result (16) is in perfect agreement with Eq. (5) for the classical apparatus function and Eq. (10) for the time-dependent Doppler effect. This confirms that the which-way interpretation which accounts for the photon recoil is indeed an equally valid physical picture for the situation in the three experiments and that it leads to the same conclusions.

Eq. (15), in analogy with Eq. (3), can also be read as the Fourier transform of the thermal wave packet,

$$V = \int_{-\infty}^{\infty} dx \, \left(\sum_n P(n)|\psi_n(x)|^2\right) \cos(2kx) \;, \tag{17}$$

where $\psi_n(x) = \langle x|n\rangle$ is the spatial representation of the n^{th} harmonic oscillator eigenfunction. Comparison with Eq. (16) confirms that the thermal spatial probability distribution is a Gaussian, as we assumed earlier.

We would like to note that the dependence of the visibility on the extension of the motional wave function, Eq. (16), implies a limited visibility also for an atom in the motional ground state. This should be experimentally observable, e.g. in the case of Fig. 1c, when the probing of the cavity field is combined with ground state cooling techniques. The dependence of the visibility on the ground state extension for different trapping strength would illustrate nicely the quantum limit of confinement of an atom, and it would be another fundamental demonstration of Bohr's moving slit experiment, similar to the work of Ref. [14].

CONCLUSIONS

We have presented several physical pictures for the probing of an optical field structure by a single trapped atom, and investigated the factors that limit the spatial resolution. We used three recent experiments as examples, where a standing wave structure was detected by a single trapped ion.

The detection of a standing wave involves the interference between the two absorption or scattering processes pertaining to the two travelling waves, therefore the spatial resolution is connected to the visibility of the interference fringes. We have given several different explanations how a limitation of the visibility arises: The spatial probability distribution of the trapped atom can be regarded to act as an apparatus function with which the ideal, full-contrast signal is convoluted. The ion's oscillation in the trap can also be considered to create periodically phase-modulated light fields, of which only the unshifted components interfere. Finally, the possible modification of the ion's motional state by the photon recoil of the two travelling waves can be considered to encode which-way information in the ion.

These seemingly different pictures lead to identical conclusions regarding their effect on the visibility, which shows that they are indeed only different interpretations of the same physical situation.

Our study is not at all limited to a standing wave. This simple case only helps to highlight the relations of more general validity, between phase modulation, spatial probability distribution, and in particular the interpretation of the photon recoil as which-way information.

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